1 TDC MTH COM (S)

2013

COMMERCE

(Speciality)

(Mathematics for Commerce)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* *(i)* If the pth, qth and rth terms of an AP are x, y and z respectively, prove that

$$p(y-z)+q(z-x)+r(x-y)=0 \qquad 3$$

(ii) If $x=2+2^{2/3}+2^{1/3}$, show that

$$x^3-6x^2+6x-2=0 \qquad 5$$

(iii) The vertices of a triangle are $A(-1, 5)$, $B(3, 1)$ and $C(5, 7)$ respectively. Find the area of the triangle formed by joining the middle points of the sides of this triangle. 4

(iv) Prove that the lines $3x-2y-4=0$ and $6x-4y-3=0$ are parallel. 2

Or

(b) (i) Find n, if ${}^{n}C_{8} : {}^{n}C_{6} = 15:28$. 3

(ii) Solve : 4

$$\sqrt{2x+1}+\sqrt{3x+4}$$

(iii) Derive an equation of a straight line passing through two given points. 4

(iv) If a straight line makes an angle 45° with the x-axis and if it passes through $(2, -1)$, find the equation of the straight line. 3

2. (a) (i) Define limit of a function. Also write down the condition for the existence of the limit of a function. 2

(ii) A function $f(x)$ is defined as

$$\begin{aligned} f(x) &= -x \quad , \text{ when } \quad x \leq 0 \\ &= x \quad , \text{ when } \quad 0 < x < 1 \\ &= 2-x \quad , \text{ when } \quad x \geq 1 \end{aligned}$$

Show that it is continuous at $x = 1$. 2

(iii) Evaluate : 3

$$\lim_{n\to\infty}\left\{\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\cdots+\frac{n}{n^2}\right\}$$

(iv) Find the maximum and minimum values of the function

$$2x^3 - 21x^2 + 36x - 20$$ 5

(v) Evaluate $\int (5^x + e^x)\,dx$. 2

Or

(b) (i) Find $\frac{dy}{dx}$, when $ax^2 + 2hxy + by^2 = 1$. 3

(ii) Evaluate : 3

$$\lim_{x \to 0} \frac{\sqrt{1+2x} - \sqrt{1-3x}}{x}$$

(iii) Find the value of

$$\int_0^1 xe^x dx$$ 2

(iv) If $y = Ae^{2x} + Be^{-2x}$, then show that

$$\frac{d^2y}{dx^2} - 4y = 0$$ 3

(v) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$ 3

3. (a) (i) Define minors and cofactors of the elements of a determinant. 2

(ii) Find the value of

$$\begin{vmatrix} 2 & 4 & 6 \\ 3 & 6 & -5 \\ 5 & 10 & 1 \end{vmatrix}$$ 1

(iii) Solve by Cramer's rule : 3

$$2x + 3y = 13$$
$$x + 7y = 23$$

(iv) If $f(x) = x^2 - 5x + 6$, find $f(A)$, if

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$ 4

(v) If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, find the values of a and b. 4

Or

(b) *(i)* When are two matrices said to be conformable for multiplication? 1

(ii) Find the product AB of two matrices A and B, where

$$A = \begin{bmatrix} 1 \\ 0 \\ 7 \\ 8 \end{bmatrix} \text{ and } B = [2 \ 4 \ 9 \ 6 \ 5 \ 0]$$ 3

(iii) Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \quad 5$$

(iv) Find the minor and cofactor of 4 in

$$\Delta = \begin{vmatrix} 2 & -1 & 1 \\ 3 & -2 & 4 \\ 1 & 1 & 2 \end{vmatrix} \quad 2$$

(v) Find the adjoint of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

and hence evaluate A^{-1}. 3

4. *(a)* *(i)* Give a general statement of a linear programming problem. What are its major components and what does the non-negativity restriction mean? 2+3

(ii) Discuss the uses of LPP in business and commerce. 4

(iii) Solve graphically the following LPP : 5

Minimize $Z = 3x_1 + 2x_2$

subject to

$$5x_1 + x_2 \geq 10$$
$$x_1 + x_2 \geq 6$$
$$x_1 + 4x_2 \geq 12$$
$$x_1,\ x_2 \geq 0$$

Or

(b) *(i)* In relation to the LP problem, explain the implications of the following assumptions of the model : 6

(1) Linearity of the objective function and constraints

(2) Continuous variable

(3) Certainty

(ii) What is meant by a feasible solution of an LP problem? 2

(iii) Use simplex method to

maximize $Z = 6x_1 + 8x_2$

subject to

$$5x_1 + 10x_2 \leq 60$$
$$4x_1 + 4x_2 \leq 40$$
$$x_1,\ x_2 \geq 0$$

6

5. *(a)* *(i)* Mantissa of logarithm of any number is always ——.

(Fill in the blank) 1

(ii) Find the base, if logarithm of 27 is 6. 2

(iii) If $a^2 + b^2 = 14ab$, prove that

$$\log\left\{\frac{1}{2}(a-b)\right\} = \frac{1}{2}\{\log 3 + \log a + \log b\}$$ 3

(iv) A merchant borrows Rs 4,800 at 6% simple interest and invests at 7½% compound interest. If the transaction is closed at the end of 3 years, what is his gain? 3

(v) A man retires at the age of 60 years and his employers gives him a pension of Rs 12,000 a year paid in half-yearly instalments for the rest of his life. Reckoning his expectation of life to be 13 years and that interest is at 4% p.a. payable half-yearly, what single sum is equivalent to his pension? 5

Or

(b) *(i)* If $2\log a + 3\log b - 2 = 0$, then prove that $a^2 b^3 = 100$. 2

(ii) Find the value of

$$\log_3\{\log_2(\log_5 25)\}$$ 1

(iii) If the population of a town increases every year by 2% of that population at the beginning of that year, in how many years will the total increase of population be 40%? 4

(iv) Define perpetuity and deferred annuity. 2

(v) A housing flat costs Rs 1,36,000, 40% to be paid at the time of purchase and balance reckoning CI @ 9% p.a. is to be paid in 12 equal annual instalments. Find the amount of each instalment. 5

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